

Objective To find and compare the measures of angles



How can you use tools like a protractor, ruler, or tracing paper to help you solve this?



Solve It! Getting Ready!

Which angles below, if any, are the same size as the angle at the right? Describe two ways you can verify your answer.

In this lesson, you will learn to describe and measure angles like the ones in the Solve It.

Essential Understanding You can use number operations to find and compare the measures of angles.



Lesson Vocabulary

- angle
- sides of an angle
- vertex of an angle
- measure of an angle
- acute angle
- right angle
- obtuse angle
- straight angle
- congruent angles

Take note

Key Concept Angle

Definition

An **angle** is formed by two rays with the same endpoint.

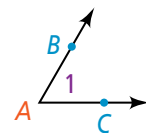
The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

How to Name It

You can name an angle by

- its vertex, $\angle A$
- a point on each ray and the vertex, $\angle BAC$ or $\angle CAB$
- a number, $\angle 1$

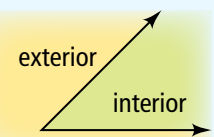
Diagram



The sides of the angle are \overrightarrow{AB} and \overrightarrow{AC} .
The vertex is A.

When you name angles using three points, the vertex must go in the middle.

The *interior* of an angle is the region containing all of the points between the two sides of the angle. The *exterior* of an angle is the region containing all of the points outside of the angle.



Think

What rays form $\angle 1$?

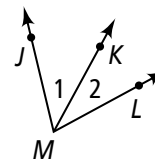
\overrightarrow{MJ} and \overrightarrow{MK} form $\angle 1$.



Problem 1 Naming Angles

What are two other names for $\angle 1$?

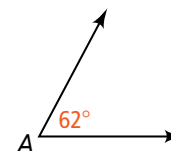
$\angle JMK$ and $\angle K MJ$ are also names for $\angle 1$.



Got It? 1. a. What are two other names for $\angle KML$?

b. **Reasoning** Would it be correct to name any of the angles $\angle M$? Explain.

One way to measure the size of an angle is in degrees. To indicate the measure of an angle, write a lowercase m in front of the angle symbol. In the diagram, the measure of $\angle A$ is 62. You write this as $m\angle A = 62$. In this book, you will work only with degree measures.

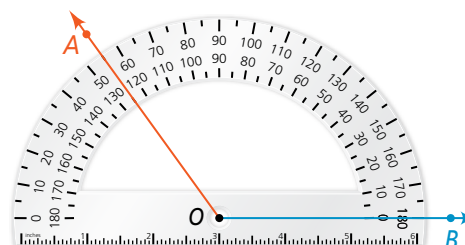


A circle has 360° , so 1 degree is $\frac{1}{360}$ of a circle. A protractor forms half a circle and measures angles from 0° to 180° .

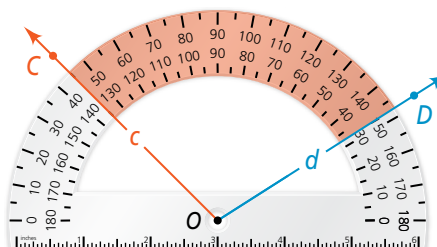
take note

Postulate 1-7 Protractor Postulate

Consider \overrightarrow{OB} and a point A on one side of \overrightarrow{OB} . Every ray of the form \overrightarrow{OA} can be paired one to one with a real number from 0 to 180.



The Protractor Postulate allows you to find the measure of an angle. Consider the diagram below. The **measure** of $\angle COD$ is the absolute value of the difference of the real numbers paired with \overrightarrow{OC} and \overrightarrow{OD} . That is, if \overrightarrow{OC} corresponds with c , and \overrightarrow{OD} corresponds with d , then $m\angle COD = |c - d|$.



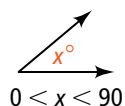
Notice that the Protractor Postulate and the calculation of an angle measure are very similar to the Ruler Postulate and the calculation of a segment length.

You can classify angles according to their measures.

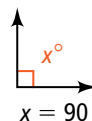
Take note

Key Concept Types of Angles

acute angle



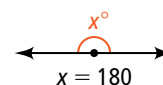
right angle



obtuse angle



straight angle

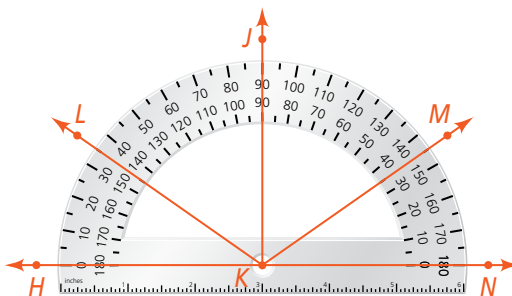


The symbol \square in the diagram above indicates a right angle.



Problem 2 Measuring and Classifying Angles

What are the measures of $\angle LKN$, $\angle JKL$, and $\angle JKN$? Classify each angle as *acute*, *right*, *obtuse*, or *straight*.



Think

Do the classifications make sense?

Yes. In each case, the classification agrees with what you see in the diagram.

Use the definition of the measure of an angle to calculate each measure.

$$m\angle LKN = |145 - 0| = 145; \angle LKN \text{ is obtuse.}$$

$$m\angle JKL = |90 - 145| = |-55| = 55; \angle JKL \text{ is acute.}$$

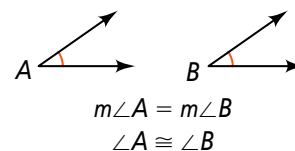
$$m\angle JKN = |90 - 0| = 90; \angle JKN \text{ is right.}$$



Got It? 2. What are the measures of $\angle LKH$, $\angle HKN$, and $\angle MKH$? Classify each angle as *acute*, *right*, *obtuse*, or *straight*.

Angles with the same measure are **congruent angles**. This means that if $m\angle A = m\angle B$, then $\angle A \cong \angle B$. You can also say that if $\angle A \cong \angle B$, then $m\angle A = m\angle B$.

You can mark angles with arcs to show that they are congruent. If there is more than one set of congruent angles, each set is marked with the same number of arcs.



Think

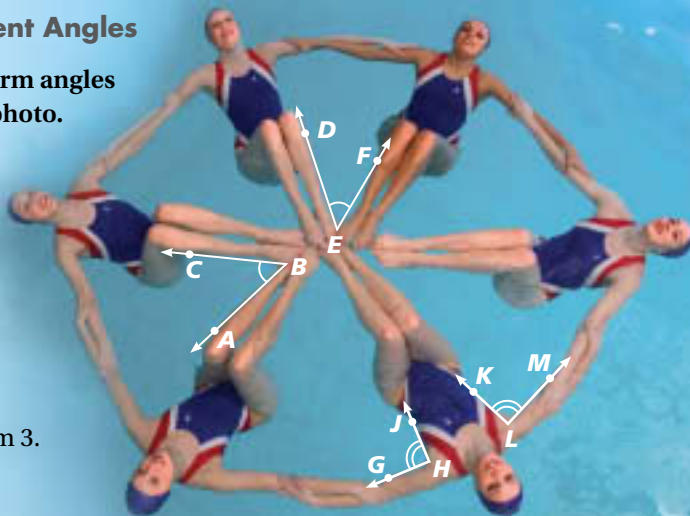
Look at the diagram. What do the angle marks tell you? The angle marks tell you which angles are congruent.



Problem 3 Using Congruent Angles

Sports Synchronized swimmers form angles with their bodies, as shown in the photo. If $m\angle GHJ = 90$, what is $m\angle KLM$?

$\angle GHJ \cong \angle KLM$ because they both have two arcs.
So, $m\angle GHJ = m\angle KLM = 90$.



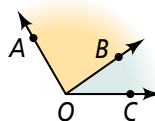
Got It? 3. Use the photo in Problem 3. If $m\angle ABC = 49$, what is $m\angle DEF$?

The Angle Addition Postulate is similar to the Segment Addition Postulate.

Take note

Postulate 1-8 Angle Addition Postulate

If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



Problem 4 Using the Angle Addition Postulate

Algebra If $m\angle RQT = 155$, what are $m\angle RQS$ and $m\angle TQS$?

$$m\angle RQS + m\angle TQS = m\angle RQT \quad \text{Angle Addition Postulate}$$

$$(4x - 20) + (3x + 14) = 155$$

Substitute.

$$7x - 6 = 155$$

Combine like terms.

$$7x = 161$$

Add 6 to each side.

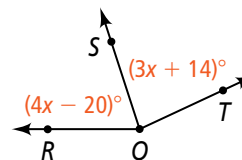
$$x = 23$$

Divide each side by 7.

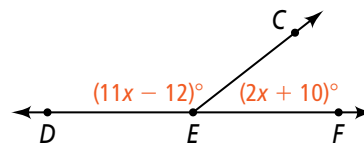
$$m\angle RQS = 4x - 20 = 4(23) - 20 = 92 - 20 = 72$$

Substitute 23 for x .

$$m\angle TQS = 3x + 14 = 3(23) + 14 = 69 + 14 = 83$$



Got It? 4. $\angle DEF$ is a straight angle. What are $m\angle DEC$ and $m\angle CEF$?



Plan

How can you use the expressions in the diagram?

The algebraic expressions represent the measures of the smaller angles, so they add up to the measure of the larger angle.

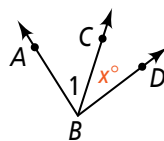


Lesson Check

Do you know HOW?

Use the diagram for Exercises 1–3.

- What are two other names for $\angle 1$?
- Algebra** If $m\angle ABD = 85$, what is an expression to represent $m\angle ABC$?
- Classify $\angle ABC$.

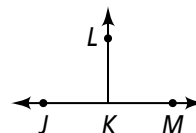


Do you UNDERSTAND?



MATHEMATICAL PRACTICES

- Vocabulary** How many sides can two congruent angles share? Explain.
- Error Analysis** Your classmate concludes from the diagram below that $\angle JKL \cong \angle LKM$. Is your classmate correct? Explain.



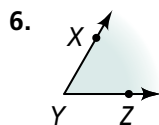
Practice and Problem-Solving Exercises



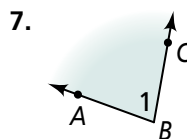
MATHEMATICAL PRACTICES

A Practice

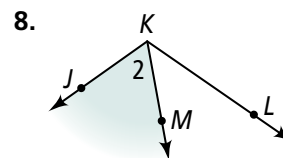
Name each shaded angle in three different ways.



6.



7.



8.

← See Problem 1.

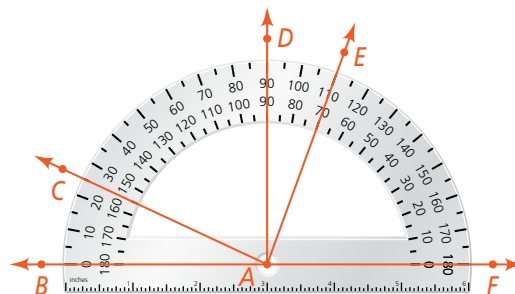
Use the diagram below. Find the measure of each angle. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

← See Problem 2.

- | | | |
|------------------|------------------|------------------|
| 9. $\angle EAF$ | 10. $\angle DAF$ | 11. $\angle BAE$ |
| 12. $\angle BAC$ | 13. $\angle CAE$ | 14. $\angle DAE$ |

Draw a figure that fits each description.

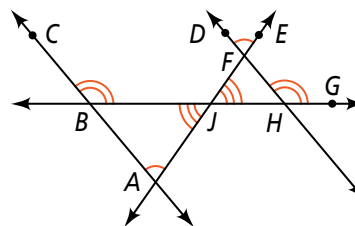
- an obtuse angle, $\angle RST$
- an acute angle, $\angle GHJ$
- a straight angle, $\angle KLM$



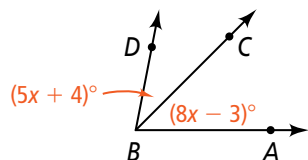
Use the diagram below. Complete each statement.

← See Problem 3.

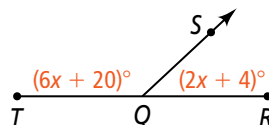
- $\angle CBJ \cong \blacksquare$
- $\angle FJH \cong \blacksquare$
- If $m\angle EFD = 75$, then $m\angle JAB = \blacksquare$.
- If $m\angle GHF = 130$, then $m\angle JBC = \blacksquare$.



22. If $m\angle ABD = 79$, what are $m\angle ABC$ and $m\angle DBC$?



23. $\angle RQT$ is a straight angle. What are $m\angle RQS$ and $m\angle TQS$?

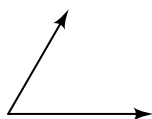


See Problem 4.

B Apply

Use a protractor. Measure and classify each angle.

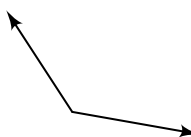
24.



25.



26.



27.



28. **Think About a Plan** A pair of earrings has blue wedges that are all the same size. One earring has a 25° yellow wedge. The other has a 14° yellow wedge. Find the angle measure of a blue wedge.

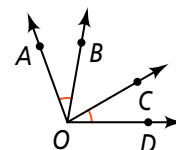
- How do the angle measures of the earrings relate?
- How can you use algebra to solve the problem?



Algebra Use the diagram at the right for Exercises 29 and 30. Solve for x . Find the angle measures to check your work.

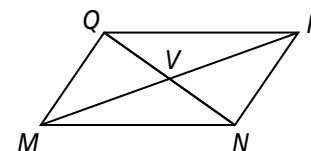
29. $m\angle AOB = 4x - 2$, $m\angle BOC = 5x + 10$, $m\angle COD = 2x + 14$

30. $m\angle AOB = 28$, $m\angle BOC = 3x - 2$, $m\angle AOD = 6x$



31. If $m\angle MQV = 90$, which expression can you use to find $m\angle VQP$?

- (A) $m\angle MQP - 90$ (C) $m\angle MQP + 90$
(B) $90 - m\angle MQV$ (D) $90 + m\angle VQP$



32. **Literature** According to legend, King Arthur and his knights sat around the Round Table to discuss matters of the kingdom. The photo shows a round table on display at Winchester Castle, in England. From the center of the table, each section has the same degree measure. If King Arthur occupied two of these sections, what is the total degree measure of his section?



C Challenge

Time Find the angle measure of the hands of a clock at each time.

33. 6:00

34. 7:00

35. 11:00

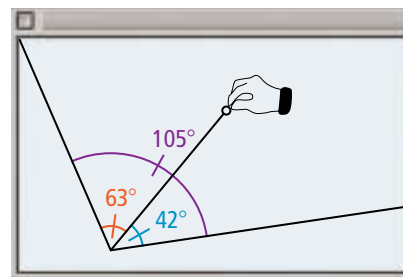
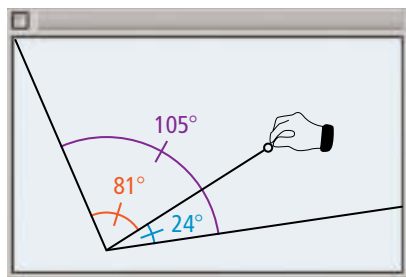
36. 4:40

37. 5:20

38. 2:15

39. **Open-Ended** Sketch a right angle with vertex V . Name it $\angle 1$. Then sketch a 135° angle that shares a side with $\angle 1$. Name it $\angle PVB$. Is there more than one way to sketch $\angle PVB$? If so, sketch all the different possibilities. (*Hint*: Two angles are the same if you can rotate or flip one to match the other.)

40. **Technology** Your classmate constructs an angle. Then he constructs a ray from the vertex of the angle to a point in the interior of the angle. He measures all the angles formed. Then he moves the interior ray as shown below. What postulate do the two pictures support?



Apply What You've Learned



**MATHEMATICAL
PRACTICES**
MP 6

Look back at the diagram on page 3 for the riddle Cameron found in an antique store. Choose from the following words and equations to complete the sentences below.

right

congruent

obtuse

$$21s + 6 = 48$$

$$21s + 6 + 48 = 90$$

$$21s + 6 + 48 = 180$$

$$s = 6$$

$$s = 2$$

$$s \approx 1.7$$

- In the riddle's diagram, $\angle HEF$ and $\angle HED$ are ? angles.
- The equation that correctly relates $m\angle HEF$ and $m\angle HED$ is ? .
- The solution of the equation from part (b) is ? .